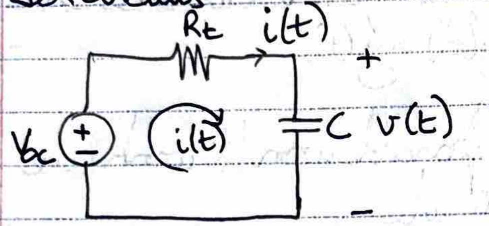
Derivation Of Response Of First Order Circuit To A Constant Input

For circuits:

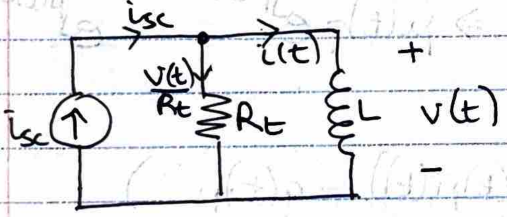


$$i(t) = C\frac{dv(t)}{dt}$$

KCL:

$$-V_{oc} + i(t)R_t + V(t) = 0 \Rightarrow C\frac{dv(t)}{dt}R_t + V(t) = V_{oc}$$

$$\Rightarrow \frac{dv(t)}{dt} + \frac{v(t)}{R_tC} = \frac{V_{oc}}{R_tC} \qquad \text{——— (1)}$$



$$v(t) = L\frac{di(t)}{dt}$$

$$i_{sc} = \frac{v(t)}{R_t} + i(t) \Rightarrow \frac{L}{R_t}\frac{di(t)}{dt} + i(t) = i_{sc}$$

$$\Rightarrow \frac{di(t)}{dt} + \frac{i(t)}{L/R_t} = \frac{i_{sc}}{L/R_t} \qquad \text{——— (2)}$$

Since (1) & (2) are similar, set 2 constants, τ and K

Let the new DE be: $\frac{dx(t)}{dt} + \frac{x(t)}{\tau} = K$.

We can solve this DE: $\mu(t) = e^{\int \frac{1}{\tau}dt} = e^{\frac{t}{\tau}}$

$\therefore x(t)e^{\frac{t}{\tau}} = \int Ke^{t/\tau} + C$.

For (1):

$\tau = R_tC$, $K = \frac{V_{oc}}{R_tC}$

For (2):

$\tau = \frac{L}{R_t}$, $K = \frac{i_{sc}}{L/R_t}$

(X) Assume K is constant:

$$\therefore x(t)e^{t/\tau} = K\tau e^{t/\tau} + c$$

$$\Rightarrow x(t) = K\tau + ce^{-t/\tau}$$

At $x=0$, $x(0) = K\tau + c$ $\Rightarrow c = x(0) - K\tau$

N.B $K\tau = V_{oc}$ for ①, I_{sc} for ② (let's say X_{TH} for this)

$$\therefore x(t) = X_{TH} + (x(0) - X_{TH})e^{-t/\tau}$$

For ①: $v(t) = V_{oc} + (v(0) - V_{oc})e^{-t/RC}$

$$i(t) = I_{sc} + (i(0) - I_{sc})e^{-Rt/L}$$

—x—