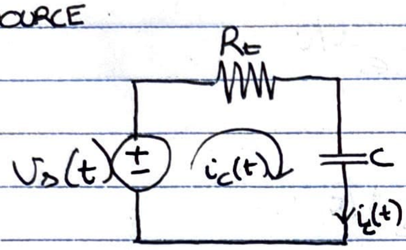
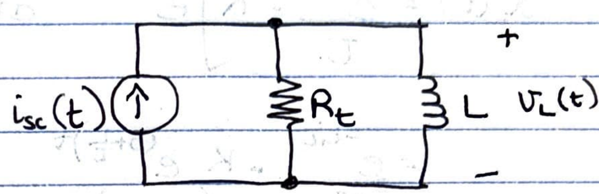
07/09/2025 DERIVATION OF RESPONSE OF A FIRST-ORDER CIRCUIT TO A NONCONSTANT SOURCE





$$i_c(t) = C\frac{dv_c(t)}{dt} \qquad v_L(t) = L\frac{di_L(t)}{dt}$$

$$-V_s(t) + R_t\left(C\frac{dv_c(t)}{dt}\right) + v_c(t) = 0 \qquad \frac{L}{R_t}\frac{di_L(t)}{dt} + i_L(t) = i_s(t)$$

$$\Rightarrow \frac{dv_c(t)}{dt} + \frac{v_c(t)}{R_tC} = \frac{+V_s(t)}{R_tC} \qquad \frac{di_L(t)}{dt} + \frac{i_L(t)}{L/R_t} = \frac{i_s(t)}{L/R_t}$$

$$(\tau = R_tC) \qquad (\tau = L/R_t)$$

Solve:

$$\mu(t) = e^{\int p(t)dt} = e^{\frac{1}{\tau}\int dt} = e^{t/\tau}$$

General form: $\frac{dx(t)}{dt} + \frac{x(t)}{\tau} = \frac{s(t)}{\tau}$

$$\therefore x(t)\mu(t) = \frac{1}{\tau}\int \mu(t)s(t)dt + C \quad \Rightarrow x(t)e^{t/\tau} = \frac{1}{\tau}\int s(t)e^{t/\tau}dt + C$$

$$\Rightarrow x(t) = \frac{e^{-t/\tau}}{\tau}\int s(t)e^{t/\tau}dt + Ce^{-t/\tau}$$

I) If $s(t)$ is a constant (M):

$$x(t) = \frac{\cancel{e^{-t/\tau}}}{\cancel{\tau}} M \cdot \cancel{\tau}\cancel{e^{t/\tau}} + Ce^{-t/\tau}$$

$$= M + Ce^{-t/\tau} \equiv x_f + x_n, \quad x_f = M, \ \underbrace{x_n = Ce^{-t/\tau}}_{\text{always remains } Ce^{-t/\tau}}$$

~~When $x_f = Ke^{bt}$~~ : When $s(t) = Ke^{bt}$

$$x(t) = \frac{e^{t/\tau}}{\tau} \cdot K \int e^{(b+1/\tau)t}\,dt + Ce^{-t/\tau}$$

$$= \underbrace{\frac{e^{-t/\tau}}{\tau} \cdot K \frac{e^{(b+\frac{1}{\tau})t}}{(b+\frac{1}{\tau})}}_{x_f} + \underbrace{Ce^{-t/\tau}}_{x_n}$$

Similarly, can generalize with lot more functions.