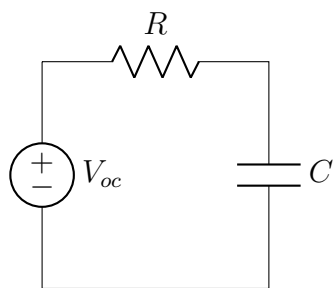


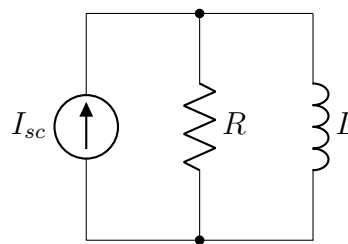
# Chapter 8: The Complete Response to RL and RC Circuits

ECE 2040

## Steps to solve first-order circuits w/ constant input



(a) RC circuit.



(b) RL Circuit.

**Figure 1.** First Order Circuits.

1. Separate the circuit into two parts - one containing just the energy storage element, and the other containing the rest of the circuit.
2. Find the steady state voltage/current across this element at  $t = 0^-$ , where an event takes place at  $t = 0$ . Usually, this involves a switch opening/closing at that specific instant.

*Remember that in steady-state, a capacitor acts as an open circuit, and an inductor acts as a short circuit.*

3. (a) If the element is an **inductor**, replace the remainder of the circuit with its **Norton** equivalent.  
(b) If the element is a **capacitor**, replace the rest of the circuit with its **Thevenin** equivalent.
4. Use the below formulae to calculate the response of the element to the given input:

$$v(t) = V_{oc} + (v(0^+) - V_{oc})e^{-t/(R_tC)}$$

$$i(t) = I_{sc} + (i(0^+) - I_{sc})e^{-t/(R_t/L)}$$

*$v(0^-) = v(0^+)$  due to the conservation of energy, capacitor voltage cannot change instantaneously. Likewise for inductor current. See Chapter 7 for more info.*

- Depending on the question, you might be asked to give the response at a specific time, or simply give the response as a function. Always make sure to understand what the question is asking.

## Sequential Switching

When a circuit contains 2+ switches that change state at different instants.

- For the first state change, follow the steps for a single switching.
- Using the response obtained in step 1, substitute the time for when the next state change happens, e.g. if a switch closes at  $t = t_0$  seconds, then find  $v(t_0^-)$ .
- Repeat the steps for single switching using these equations instead:

$$v(t) = V_{oc} + (v(t_0^+) - V_{oc})e^{-(t-t_0)/(R_t C)}$$

$$i(t) = I_{sc} + (i(t_0^+) - I_{sc})e^{-(t-t_0)(R_t/L)}$$

*Basically, we are shifting the reference point of above equation which assumes  $t_0 = 0$ .*

- Repeat steps 2-3 for further switching at different instants.

## Steps to solve first-order circuits w/ nonconstant input

**TABLE 1**  
POSSIBLE FORCED RESPONSES FOR INPUT FUNCTIONS

Forcing function, $y(t)$	Forced response, $x_f(t)$
$y(t) = M$	$x_f = N$
$y(t) = Me^{-bt}$	$x_f = Ne^{-bt}$
$y(t) = M \sin(\omega t + \theta)$	$x_f = A \sin(\omega t) + B \cos(\omega t)$

- Guess the forced response for the circuit based on the forcing function (i.e. source voltage/current).
- Write down the differential equation for the circuit.
- Substitute the forced response  $x_f$  for the  $v_c/i_l$ , and solve for the constant.
- The natural response will be of the form  $x_n = Ce^{-t/\tau}$ . Now, write down the complete response  $x = x_n + x_f$ .
- Find  $C$  by substituting the steady-state value in  $x(t)$ .