Chapter 8: The Complete Response to RL and RC Circuits

ECE 2040

Steps to solve first-order circuits w/ constant input

(a) RC circuit. **(b)** RL Circuit.

Figure 1. First Order Circuits.

1. Separate the circuit into two parts - one containing just the energy storage element, and the other containing the rest of the circuit.
2. Find the steady state voltage/current across this element at $t = 0^-$, where an event takes place at $t = 0$. Usually, this involves a switch opening/closing at that specific instant.

   Remember that in steady-state, a capacitor acts as an open circuit, and an inductor acts as a short circuit.
3. (a) If the element is an **inductor**, replace the remainder of the circuit with its **Norton** equivalent.
   (b) If the element is a **capacitor**, replace the rest of the circuit with its **Thevenin** equivalent.
4. Use the below formulae to calculate the response of the element to the given input:

$$v(t) = V_{oc} + (v(0^+) - V_{oc})e^{-t/(R_t C)}$$

$$i(t) = I_{sc} + (i(0^+) - I_{sc})e^{-t(R_t/L)}$$

   $v(0^-) = v(0^+)$ *due to the conservation of energy, capacitor voltage cannot change instantaneously. Likewise for inductor current. See Chapter 7 for more info.*

5. Depending on the question, you might be asked to give the response at a specific time, or simply give the response as a function. Always make sure to understand what the question is asking.

Sequential Switching

When a circuit contains 2+ switches that change state at different instants.

1. For the first state change, follow the steps for a single switching.
2. Using the response obtained in step 1, substitute the time for when the next state change happens, e.g. if a switch closes at $t = t_0$ seconds, then find $v(t_0^-)$.
3. Repeat the steps for single switching using these equations instead:

$$v(t) = V_{oc} + (v(t_0^+) - V_{oc})e^{-(t-t_0)/(R_t C)}$$

$$i(t) = I_{sc} + (i(t_0^+) - I_{sc})e^{-(t-t_0)(R_t/L)}$$

   Basically, we are shifting the reference point of above equation which assumes $t_0 = 0$.

4. Repeat steps 2-3 for further switching at different instants.

Steps to solve first-order circuits w/ nonconstant input

Table 1
Possible forced responses for input functions

Forcing function, $y(t)$	**Forced response,** $x_f(t)$
$y(t) = M$	$x_f = N$
$y(t) = Me^{-bt}$	$x_f = Ne^{-bt}$
$y(t) = M\sin(\omega t + \theta)$	$x_f = A\sin(\omega t) + B\cos(\omega t)$

1. Guess the forced response for the circuit based on the forcing function (i.e. source voltage/current).
2. Write down the differential equation for the circuit.
3. Substitute the forced response x_f for the v_c/i_l, and solve for the constant.
4. The natural response will be of the form $x_n = Ce^{-t/\tau}$. Now, write down the complete response $x = x_n + x_f$.
5. Find C by substituting the steady-state value in $x(t)$.